Chapter 1

Wolfgang Vogel: reminiscences of a mathematical friendship

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I spent the 1976 academic year visiting the I.H.E.S., thoroughly enjoying the French for their mathematics, culture, and good life. About half way through that year I was invited by Wolfgang Vogel, whom I had never met, to come to Halle, in East Germany, to visit, give a lecture, and discuss mathematics. Contacts with mathematicians from the East were then few and difficult. East Germany seemed, by contrast with Paris, a grey, depressed place with bad air and unfriendly border guards, not a desirable tourist stop, but I felt that Vogel's invitation was an effort to bridge the gap, and I was eager to accept. A refugee from neighboring Leipzig when she was five, my wife hadn't been back since then and came along. The trip seemed exotic and slightly risky.

Wolfgang, the best of hosts, met us in Leipzig, where we had to change from a major train line to the local system. He too felt that such a trip was a little perilous, and was anxious that we have a smooth experience. I clearly remember the first breakfast we had with him, near the Leipzig station: we spoke easily and got on from the first, and the conversation ranged widely over mathematics that interested us. I expected that Wolfgang would be guarded in what he said about the local political situation; but I was immediately comfortable with him, and he was not so careful as to avoid all political references: he jokingly told

me that "We in the East have an important research advantage over your in the west: we know exactly what theorems we are going to prove five years in advance!" Each time I write an NSF proposal I think of those words.

We had plenty of adventures that first visit! On the train from Leipzig to Halle my wallet—with my passport—mysteriously disappeared from my coat, a loss that we discovered as soon as I tried to check in at the University guest house. I was cavalier about this, not understanding the significance of the event, but Vogel was horrified. We went straight to the police, where we were assured that because of the great honesty (and strong police) of the East German people my passport would surely be recovered within hours (it never was). At this point my good luck returned: the US had just re-opened its consulate in Berlin, and although it was impossible to call information and simply ask for the phone number, Vogel had a friend who knew it. We traveled together to Berlin to visit the consulate. It might have been compromising for Vogel to come near the building, but he took me to within a couple of blocks and pointed the way. It turned out that the Consul had studied at the University of Chicago at almost the same time I had. After discussing our mutual acquaintances he told me that if I was a spy I was an awfully good one, and wrote me out a new passport (I was detained at the border anyway, by guards who were incredulous when they saw a US passport issued in East Berlin!)

With this adventure, Wolfgang and I became much closer. Walking out of doors he would speak directly of the oppressive regime in which he lived. For example, he told me, that any letter referring to technical matters HAD to be sent from the University; and that any such letter had to be given for prior approval, with a copy, to the authorities before it was sent. They would first consider whether it was "in Ordnung"; and they would keep the copy on file afterwards, "In case a question arose." He also told me of his intense wish to make something of the mathematical life in Halle, of his efforts to establish and hold connections with others outside of East Germany, and of his struggles to build a mathematical group based on good research, and not on acceptable politics. These were themes that we discussed over many years. After that first visit we met every year or two. Mostly I came to Halle, but in periods when he was allowed to travel (which for ten years he was not, after his wife's brother defected) we met at conferences, and I also brought him to Brandeis on a couple of occasions.

Early in our acquaintance I had the pleasure of getting to know Wolfgang's wife Dagmar, whom I found extremely open, lively, and attractive, as well as proud of her husband and eager to make the conditions good for his work. A medical technician, she told me that when she first met Wolfgang, at a medical students' ball in 1962, she thought to herself, "A strange bird (=Vogel) worth getting to know!" Later she wrote, "Nie hatte ich zuvor einen Studenten getroffen, der so zielstrebig arbeitete. Aber schon in dieser frühen zeit hatte er dieses Feuer, diese liebe zur Mathematik in sich. Dies übertrug sich auf mich, so dass ich (fast) immer Verständnis aufbrachte." (Never before had I met a student so strongly directed toward his goal. Already in this early time he had that fire, that love for mathematics. This rubbed off on me, so that I was (almost) always

sympathetic.")

Wolfgang made sure that my visits to Halle were very pleasant. They all followed a certain pattern: Once I had arrived and checked into the University Guest-House (it was too dangerous for him to have a western guest sleep at his house) we would go out to a cafe or Weinstube (depending on the hour), and later to his apartment, to discuss mathematics. His rich agenda of interesting problems made up the backbone of our three or four days together. There were some breaks, though. Once when I arrived particularly bleary after a long flight Wolfgang asked me whether I'd like to go to sit in a coffee shop and I said that what I'd really like was to be outdoors for a while, perhaps even to go running. Wolfgang immediately brightened: he had resolved to give up his daily run to entertain me. Soon, with Dagmar, we were piled in his car on the way to the forest at the outskirts of Halle. They loved running in pretty countryside, feeling free outside the city, and and did it daily (perhaps because of this motivation, they were much stronger runners than I).

On these visits I always spoke at Wolfgang's seminar, which was attended by an interesting cast of characters. There was lots of discussion, and after the seminar the group would go out to eat together. I got to know the people who were his protegés and collaborators, particularly Jürgen Stückrad, Trung, and Hoa. Vogel was very much the benign father of this group, for whom he had fashioned a bit of mathematical haven in the midst of that highly political world.

There was another invariable feature of these visits: when the weekend came (or if necessary before!) we would pile into the train (later into his tiny, balky car) and set forth to see some of the culture of the area. I especially remember visits to Goethe's Weimar and to Bach's Eisenach. Dagmar usually came along on these trips. Both Wolfgang and Dagmar were well-read and interested in the local history, and I was their rapt (but not very apt) pupil. Wolfgang loved classic philosophy (Nietsche, Hegel, Kant, Jaspers) and literature (Sartre, Beckett, Joyce, Proust, Grass, Zweig, Mann, Freud), and knew the art museums well. We discussed current politics too, though only when safely private: at that period, I am told, one in seven East Germans was employed by the police. But the topic on which we spent most of our energy was mathematics. We always traveled with what we jokingly referred to as the "Handbibliothek"— an enormous briefcase preloaded with all the books and journal articles that Wolfgang thought we might need for the ongoing discussion. On trains and platforms and in hotel lobbies, these materials and the problems got a good workout. They were exciting, intense times.

Wolfgang came from a working-class family that had lived for many generations in Bremen. Born in 1940 he lived in poverty in the war years, and his parents house was completely destroyed by bombs near the war's end. I do not know how Wolfgang first came to the idea of studying mathematics, but his first teaching experience came early around the time he was 14: during a teacher's two-week absence that year he was elected by the class to be the substitute (I'm told he assigned problems that were too hard!) In the University he became the student of the algebraic geometer Ott-Heinrich Keller, then

Director of the Mathematical Institute at Halle, and of Keller's assistant Bodo Renschuch. His Diplomarbeit (master's thesis) in 1963 set the direction for his whole mathematical life: it was entitled "Zur idealtheoretischen Ordnungsbestimmung von Schnittmannigfaltigkeiten" (On the ideal-theoretic determination of intersection multiplicities). After his PhD in Halle he was a postdoctoral student with Wolfgang Gröbner, in Innsbrück.

It is worth saying a few words about the background of Wolfgang's interest in intersection theory; Flenner has described some of the fruit elsewhere in this volume (pages 17–25). Intersection theory has been one of the central concerns of algebraic geometry since the second half of the 19th century. The intersections of plane curves were first properly understood at that time. Early in this century, as ideal theory developed in commutative algebra, such intersections came to be expressed in terms of ideals: the intersection multiplicity of two curves, without components in common, defined in the complex plane by polynomial equations $f(x,y) = 0$ and $g(x,y) = 0$ at the origin (0,0) is the length of the local ring $\mathbf{C}[[x,y]]/(f,g)$. This definition is "correct" in the sense that it makes Bézout's theorem (a generalization of the statement that a polynomial in one variable of degree d has d roots) true.

A major preoccupation of algebraic geometry in this century has been to generalize this simple relation. Wolfgang Gröbner suggested that if two subvarieties defined by ideals I and J in a smooth variety meet in a point with local ring R then (generalizing the relation above directly) the multiplicity should be the length of the ring $R/I + J$. Unfortunately this relation leads to a violation of Bézout's theorem. Weil proved the existence of multiplicities with the correct properties, and Serre subsequently gave a formula in which the length of $R/I + J = \mathrm{Tor}_0^R(R/I, R/J)$ is "corrected" by the alternating sum of the lengths of the other $\mathrm{Tor}_i^R(R/I, R/J)$.

It was this circle of ideas in which Wolfgang was interested from the first. One of his goals was to give a theory that would also work when the varieties do not intersect in the correct dimensions. Another was to give a definition in terms of lengths of factor rings R/K alone. He finally achieved both these goals in the mid-eighties (see for example entry [84-3] in the Vogel Bibliography, page 7). I think that this achievement of Vogel's was the one of which he was proudest.

There was an interesting twist in the story, because Fulton had in the meantime given a very general theory in terms quite different than the lengths of factor rings, and his theory was known to lead to good results. (Fulton's theory was aimed at intersections within arbitrary smooth varieties, not just in projective space, and yields an invariant finer than a multiplicity in these cases.) A certain degenerate example, whose multiplicities were computed in Fulton's very well-known book, came out differently in Vogel's theory, and many people, myself included, considered this a fatal flaw. Vogel had the last laugh: in the second edition of Fulton's book, a computational error is corrected and the new multiplicities agree with Vogel's! Since then Fulton and others have shown that the two theories lead to equivalent results in the case where they both apply.

Wolfgang traveled often to other East Bloc countries, and worked hard to

help the groups in commutative algebra in Hungary and elsewhere. He was eager to help me visit these countries too, and suggested trips to Poland and to Hungary which proved very productive: In Poland I met Jerzy Weyman, while in Hungary I was assigned, as indispensable guide, János Kollár; I was able to help both these wonderful students come to Brandeis. I doubt they ever knew that Wolfgang had something to do with their coming to the West!

Among the more distant groups in commutative algebra that Wolfgang fostered was the one in Vietnam: his first students from there came by accident, but once the pioneers found such a good mentor they sent more, so that Vogel finally had four Vietnamese PhD students, Cuong, Hoa, Nhi, and Trung, now all strongly active there. Trung describes some of his experience as a student on pages 35–38 in this volume. The conference from which this book comes was ultimately a result.

After the fall of Berlin wall and the unification of Germany there was a reaction against the people who had been influential in the East German Universities. As I understand it, the reaction had different severity in different parts of Germany, and was particularly severe in Halle. Vogel had been one of the most active and distinguished of the mathematicians in his department, and had been chairman. All the professors had been party members; it was a condition of the job there. In his fight to keep mathematics going, he had made many enemies. Although I have no access to the primary sources, I am convinced from my long knowledge of him that he tried hard to act honorably and had a very positive influence. Many friends in and out of East Germany fought to save his position, but in the end he was deemed to have been "too close to power," and was forced to leave. Naturally, Wolfgang was extremely bitter about this exclusion, but he rallied his energy, and did not despair. After a year in the United States he found a professorship at Massey University in New Zealand, where he began again; his inaugural address there is reprinted on pages 145–154 of this volume. He was very proud of the fact that he already had students and postdocs at Massey, and looked forward to building up a school of mathematics there. (I looked forward to renewing the tradition of coming to visit, too!)

Wolfgang died in the Fall of 1996 after a relapse of the cancer with which he had battled a year and a half before. In March of 1995, shortly before becoming ill for the first time, he wrote:

> Je länger ich lebe, um so mehr wende ich mich von Deutschland ab. Ich danke Gott, daß er mich nach Neuseeland geführt hat. Am Ende der Welt zu sein, ist wirklich ein Geschenk, obwohl es eine Zeit dauert, das zu begreifen. Aber Neuseeland ist tief in mir. Was würde ich nicht alles geben, hier in Frieden sterben zu können.

> (The longer I live, the more I turn away from Germany. I thank God that he led me to New Zealand. To be at the end of the world is good fortune, though it took a while to grasp this. But New Zealand is deep within me. What would I not give to be able to die in peace here.)

Above all I remember Wolfgang's intense involvement with mathematics, and his pleasure in sharing it. He often closed his letters with a salutation, drawn, I believe, from Goethe: "Frohes Schaffen" (which one might translate as "happy creating"), Wolfgang. He had the pleasure of creation deeply, and he sincerely wished it for others. I shall always remember him for this.