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The monodromy of Weierstrass points

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Summary. We show that the monodromy of the family of curves (Riemann surfaces) acts as the full symmetric group on the Weierstrass points of a general curve. The proof uses a degeneration to certain reducible curves, and the theory of limit series developed in our (1986, 1987a, b). Some of the monodromy is actually constructed by fixing a (reducible) curve and varying its "canonical" series.

Introduction

If $\pi\colon X \to Y$ is a generically finite map of varieties over $\mathbb{C}$, then the *monodromy group* of π is the image of the fundamental group of the open set of regular values of π in the group of permutations of the (finite) set of points in the general fiber of π under the map obtained by analytic continuation. It is a birational invariant of X, Y, π, and may be identified with the Galois group of π; that is, the group of automorphism of the normal closure L of the field $K(X)$ of rational functions over the field $K(Y)$, acting on the maximal ideals of $L \bigotimes_{K(Y)} \overline{K(Y)}$, where $\overline{K(Y)}$ is the algebraic closure of $K(Y)$ [see for example Harris (1979)].

Let $\mathcal{M}_g$ be the moduli space of curves of genus g, $\mathcal{C}_g \to \mathcal{M}_g$ be universal curve (that is, the moduli space of pointed curves), $W \subset \mathcal{C}_g$ the locus of Weierstrass points, and

$$\pi\colon\ W \to \mathcal{M}_g$$

the corresponding projection. We will prove:

Theorem. *The monodromy group of π is the full symmetric group on the Weierstrass points.*

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For $g=0, 1$ there is of course nothing to prove. For $g=2$ the result follows easily from the construction of genus 2 curves as double covers of $\mathbb{P}^1$. Similarly, for $g=3, 4$ the theorem has been proved by analyzing the canonical map. A general curve of genus 3 for example is a plane quartic, and its Weierstrass points are its flexes. The monodromy of the flexes of a plane *cubic* was treated already by Camille Jordan (1870) and the computation was extended to plane curves of all degrees in the paper of Harris (1979); in particular, the case $g=3$ of the theorem above is proved there. For $g=4$ the general canonical curve is the complete intersection of a quadric and a cubic in $\mathbb{P}^3$, and the Weierstrass points are its stalls; using this canonical embedding, Canuto (1979) proved that the monodromy group of the Weierstrass points is the full symmetric group for $g=4$.

As g becomes larger, the canonical embedding becomes more complex and its geometry less well understood, so that the approach used for $g=2, 3, 4$ does not seem to extend.

The proof given in Sect. 2, which applies uniformly for $g \geqq 4$ (or a variant sketched in Sect. 3 which applies for $g \geqq 5$) is based instead on the theory of *limit series*, and in particular *limit canonical series* developed in our papers (1986, 1987a, b).

1. Preliminaries

We recall some necessary definitions and notation. Basic references are our (1986, 1987a, b).

We work throughout with complete, reduced, connected curves over $\mathbb{C}$ whose singularities are ordinary double points.

If Y is a smooth curve, then a g_d^r on Y is a pair $L=(\mathscr{L}, V)$ where $\mathscr{L}$ is a line bundle of degree d and $V \subset H^0(Y, \mathscr{L})$ is an $r+1$-dimensional vectorspace. The *ramification sequence* $\alpha^L(p)$ of L at a point $p \in Y$ is the sequence $\alpha^L(\mathrm{p})$: $0 \leqq \alpha_0^L(0) \leqq \ldots \leqq \alpha_r^L(p) \leqq d-r$ defined by the condition that for $i=0, \ldots, r$, V contains a section vanishing to order precisely $\alpha_i^L(p)+i$ at p.

If L is the *complete canonical series* $(K_Y, H^0(Y, K_Y))$ then we say that p is a Weierstrass point if $\alpha^L(p) \neq (0, \ldots, 0)$, and then $\alpha^L(p)$ is the *type* of p.

A Weierstrass point $p \in Y$ is *dimensionally proper* if in the space of pointed curves near (Y, p), the Weierstrass points of the same type form a family of codimension equal to the *weight* $\Sigma \alpha_i^L(p)$. If

$$\alpha: 0 \leqq \alpha_0 \leqq \ldots \leqq \alpha_r \leqq d-r$$

is a sequence of integers, then

$$G_d^r(Y, (p, \alpha))$$

is defined to be the family of g_s^r's L on Y with ramification sequence $\alpha^L(p)$ termwise $\geqq \alpha$; and $\mathring{G}_d^r(Y, (p, \alpha))$ is the open set with $\alpha^L(p)=\alpha$; we extend these notations in an obvious way to the case of several points and ramification conditions.

A curve has *compact type* if its components are smooth and its dual graph is a tree. A *crude limit* g_s^r or *crude limit series* on a curve C of compact type is a collection

$$L = \{L_Y = (\mathscr{L}_Y, V_Y) \mid Y \text{ a component of } C\}$$

where L_Y is a g_d^r on Y, called the Y-aspect of L; that is, $\mathscr{L}_Y$ is a line bundle of degree d, and V_Y is an $r+1$ – dimensional vector subspace of $H^0(Y, \mathscr{L}_Y)$, satisfying the following compatibility condition:

If Y and Z are components meeting in p then

$$\alpha_i^{L_Y}(p) + \alpha_{r-i}^{L_Z}(p) \geqq d - r \quad \text{for} \quad i = 0, \ldots, r;$$

in case equality holds for each i we say that L is a *refined limit series*, or simply a *limit series*. Some limit g_d^r's on C arise from families of g_d^r's on smooth curves degenerating to C in such a way that their ramification points tend to smooth points of C [(1986), Sect. 2]; a limit series is called *smoothable* if it arises in this way. A smooth point $p \in Y \subset C$ of C is a ramification point of L if it is a ramification point of the Y-aspect L_Y.

A limit canonical series on C is simply a limit g_{2g-2}^{g-1} *on* C. It is proven in Sect. 4 of our (1986) that the underlying line bundle of the Y-aspect of a limit canonical series on C, for any component Y, is the bundle $K_Y(2\Sigma g_i p_i)$, where p_i runs over the nodes of C on Y and g_i is the genus of the connected component the closure of $C - Y$ containing p. However, the spaces of sections associated to the limit canonical series need not be unique, nor need every limit canonical series be smoothable. Nevertheless, all limit canonical series (and indeed all limit series) occurring in this paper will be smoothable.

If C is a curve of compact type and L a limit canonical series, then a ramification point for L will be called a Weierstrass point of C (with respect to L, if this is not clear from context).

2. The proof for $g \geqq 4$

Because the monodromy is a birational invariant, we may compute it over any irreducible (partial) compactification of $\mathscr{M}_g$. In what follows, we essentially work with the partial compactification which is the "moduli space of smoothable limit canonical series on curves of compact type". To avoid the foundational problems posed by the above words, however, we prefer to give families of smoothable limit canonical series inducing enough monodromy transformations and then think of approximating the paths that give the monodromy in the given families by paths that stay "just" inside $\mathscr{M}_g$.

The families and configurations of curves of which we will make use are summarized in Fig. 2.1 i)–v). We will use these families to check that the monodromy group G fulfills the hypothesis i) and ii) of the following elementary lemma, whose proof we leave to the reader:

Lemma 2.1. *Let G be a group of permutations of a set $W = U_1 \dot{\cup} \ldots \dot{\cup} U_g$.* If

i) *G contains permutations interchanging the subsets U_i doubly transitively; and*

ii) *For some subset* $U'_1 \supsetneqq U_1$, *G contains the group of all permutations of W fixing* $W - U'_1$ *pointwise;*
then G is the full symmetric group on W. □

We now describe the families and configurations of curves we will use. Start with a smooth projective curve of genus g; this may be degenerated to a reducible curve consisting of a curve C_{g-1} of genus $g-1$ with an elliptic tail E_1 attached at a point p_1 which is *not* a Weierstrass point of C_{g-1}, as in Fig. 2.1 i):

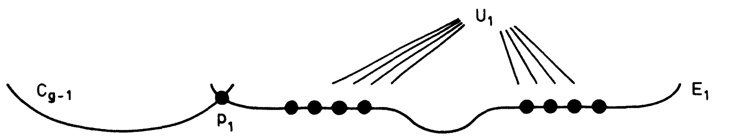

Fig. 2.1 i)

Here U_1 denotes the set of g^2-1 points on E_1 other than p_1 differing from p_1 by torsion of order g; these are the Weierstrass points of the union $C_{g-1} \cup E_1$.

Next, we can degenerate C_{g-1} to a copy of $\mathbb{P}^1$ with $g-1$ elliptic tails $E_2, \ldots, E_g$ attached at points $p_2, \ldots, p_g$, as in Fig. 2.1 ii):

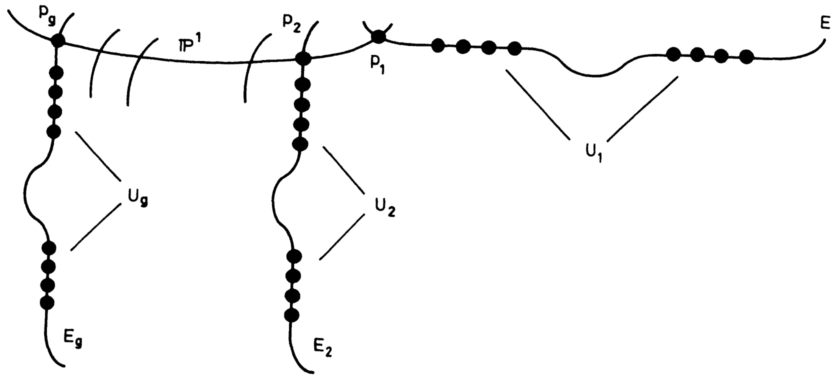

Fig. 2.1 ii)

The Weierstrass points of $\mathbb{P}^1 \cup E_1 \cup \ldots \cup E_g$ are the union of the g sets $U_1, \ldots, U_g$; and clearly the monodromy of g points on $\mathbb{P}^1$ acts on the Weierstrass points of this curve, preserving the decomposition into sets U_i and permuting the g sets U_i as the full symmetric group on g letters.

Return now to the curve pictured in Fig. 2.1 i). We let C_{g-1} specialize to a curve C'_{g-1} so that p_1 becomes a Weierstrass point of the curve C_{g-1}, and then specialize further so that p_1 becomes a Weierstrass point of type $(0, \ldots, 0, 2)$ on C_{g-1}. Under the first specialization one of the Weierstrass points of the union $C_{g-1} \cup E_1$ lying on C_g – call it x_1 – tends to p_1, so that the limit of this family in the moduli of pointed curves is a three component curve, with a rational curve inserted in between C_{g-1} and E_1 (and meeting C_{g-1} at p_1) and x_1 tending to a point of this middle component. Similarly, under the second specialization a second Weierstrass point of the union $C_{g-1} \cup E_1$, which we will denote by x_2, tends to p_1; the resulting stable pointed curve is pictured in Fig. 2.1 iii):

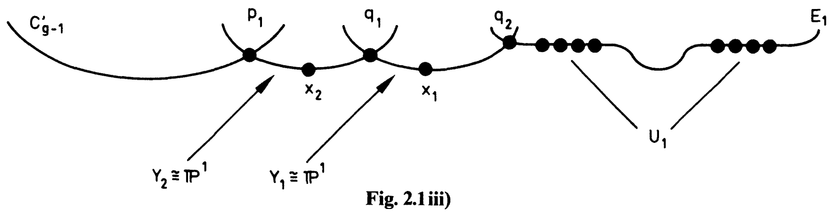

Fig. 2.1 iii)

Here the limiting positions of the Weierstrass points are, as indicated, the points of U_1, plus x_1 and x_2, plus $(g^2-1)(g-1)-2$ points of C'_{g-1}; we denote the new nodes of this curve by q_1 and q_2.

Next, we want to smooth this curve in two different ways in turn. First, we smooth the node q_1 of this curve, creating a curve as pictured in Fig. 2.1 iv), with Weierstrass points as marked:

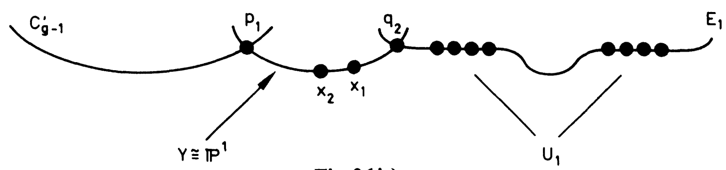

Fig. 2.1 iv)

(Here the points marked U_1 in the diagram are deformations of the points of order g on E_1; they need no longer be exactly those points.) We will show below that the monodromy of the four marked points of the component Y transposes the points x_1 and x_2, fixing all the other Weierstrass points.

Second, we want to go back to the curve of Fig. 2.1 iii) and smooth the point q_2. We arrive then at the curve pictured in Fig. 2.1 v):

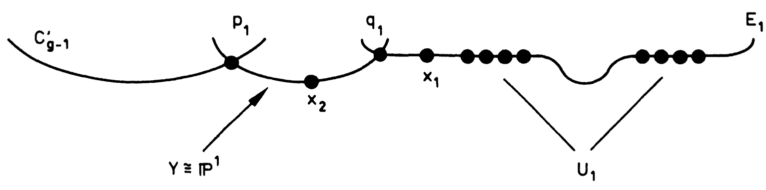

Fig. 2.1 v)

The curve E_1 will thus contain g^2 of the Weierstrass points of the union, the g^2-1 points of U_1 plus a deformation of x_1. As in the last case, the points marked U_1 are Weierstrass points of the union – that is, inflectionary points of a smoothable limit canonical series – but not necessarily the points of order g on E_1. Indeed, we will show below that $U_1 \cup \{x_1\}$ is an arbitrary fiber of a base point free pencil on E_1, and thus is permuted transitively by the monodromy of the pencil.

Using the last two families, we see that the monodromy on the Weierstrass points of the curve of Fig. 2.1 iii) contains all permutations stabilizing the complement of the set $U_1 \cup \{x_1, x_2\}$; so that the conditions of Lemma 2.1 will be satisfied.

It is of course essential that all the limit canonical series to be constructed are smoothable. This follows easily, as the reader may check for himself, using dimensional properness, from Corollary 3.7 of our (1986); see also Sect. 1 of (1987b).

The argument summarized in Fig. 2.1 i)–ii) which leads to condition i) of Lemma 2.1, is easy: when a curve of genus g degenerates to a rational curve with g elliptic tails E_i as in Fig. 2.1 i), then since such a curve is a residually generic, the Weierstrass points must go in groups U_i of g^2-1 points to the points on the E_i which differ from the attachment points p_i by g-torsion [our (1987a), Sect. 3]. Evidently the monodromy of g-tuples of attachment points on $\mathbb{P}^1$ gives the action required by Lemma 2.1, i). (We have inserted the step of degenerating to $C_{g-1} \cup E_1$ in order to isolate U_1, for use in the next argument.)

The argument summarized in Fig. 2.1, iii)–v) is more delicate. We restrict ourselves now to $g \geqq 4$. The first point to establish is the existence of C'_{g-1}.

Proposition 2.2. *There exist dimensionally proper Weierstrass points of type* (0, ..., 0, 2) *on curves of genus k so long as $k \geqq 3$.*

Proof. For $k \geqq 4$ this is guaranteed by Theorem 1 of our (1987a). For $k=3$ we are asserting that hyperflexes occur in the family of pointed smooth plane quartics in codimension 2, and this is easy to verify directly by counting dimensions. □

The degeneration indicated from $C_{g-1} \cup E_1$ to $C'_{g-1} \cup Y_1 \cup Y_2 \cup E_1$ is best constructed by constructing a limit canonical series L on the second curve and partially smoothing it. We define L by taking its aspect g^{g-1}_{2g-2}'s as follows:

$L_{C'_{g-1}}$ is $|K_{C'_{g-1}}+2p_1|$ (the complete series associated to the canonical bundle on C'_{g-1} twisted by 2 points)

we have

$$\alpha^{L_{C'_{g-1}}}(p_1)=(0, 1, \ldots, 1, 3).$$

L_{Y_2} is the unique series on $Y_2 \cong \mathbb{P}^1$ with ramification

$$\alpha^{L_{Y_2}}(p_1)=(g-4, g-2, \ldots, g-2, g-1)$$

$$\alpha^{L_{Y_2}}(x_2)=(0, \ldots, 0, 1)$$

$$\alpha^{L_{Y_2}}(q_1)=(0, 1, \ldots 1, 2).$$

L_{Y_1} is the unique series on $Y_1 \cong \mathbb{P}^1$ with ramification

$$\alpha^{L_{Y_1}}(q_1) = (g-3, g-2, \ldots, g-2, g-1)$$
$$\alpha^{L_{Y_1}}(x_1) = (0, \ldots, 0, 1)$$
$$\alpha^{L_{Y_1}}(q_2) = (0, 1, \ldots, 1).$$

L_{E_1} is $|g q_2| + (g-2) q_2$, the complete series associated to $\mathcal{O}_{E_1}(g q_2)$, plus $g-2$ base points at q_2. The ramification of L_{E_1} is

$$\alpha^{L_{E_1}}(q_2) = (g-2, \ldots, g-2, g-1).$$

The idea is as follows: The aspect on C'_{g-1} is forced. The aspect on E_1 is determined by a wish to make it the same as the E_1-aspect of the unique limit canonical series on $C_{g-1} \cup E_1$, computed in our (1987a), Sect. 3. Finally, the aspects on Y_1 and Y_2 are chosen in the unique way that makes x_1 and x_2 into ramification points and which satisfies the compatibility conditions for limit series; their existence, and ultimately uniqueness, is guaranteed by our (1983), Theorem 2.3. By hypothesis $L_{C'_{g-1}}$ is dimensionally proper with respect to p_1, so since every series on a rational curve is dimensionally proper, the restriction of L to $C'_{g-1} \cup Y_2 \cup Y_1$ can be smoothed preserving the ramification at q_2. Attaching E_1, with the constant family of aspects $|g q_2| + (g-2) q_2$, we get the desired family connecting $C_{g-1} \cup E_1$ to $C'_{g-1} \cup Y_2 \cup Y_1 \cup E_1$.

Starting from $C'_{g-1} \bigcup_{p_1} Y_2 \bigcup_{q_1} Y_1 \bigcup_{q_2} E_1$, if we smooth q_1, keeping the C'_{g-1}-aspect and E_1-aspect of L constant, we arrive at a curve $C'_{g-1} \cup Y \cup E_1$ with a limit canonical series whose Y-aspect is the unique g^{g-1}_{2g-2} on Y having the same ramification at p_1, x_2, x_1, q_2 as the linear series on $Y_1 \bigcup_{q_1} Y_2$ whose aspects on L_{Y_1} and L_{Y_2} above. (This smoothing corresponds to the family of stable 4-pointed rational curves obtained by letting x_1 and x_2 come together on Y.) But by Theorem 2.3 of our (1983), there is a unique such series for *any* relative position of the x_1, x_2 with respect to p_1 and q_2 in Y, and this family of linear series on $C'_{g-1} \bigcup_{p_1} Y \bigcup_{q_2} E_1$ gives rise to monodromy which acts by transposing x_1 and x_2, keeping all the other Weierstrass points fixed. (Note that here – as in the next family to be considered – what is varying is the limit canonical series, not the curve!)

In order to produce monodromy inducing all the permutations of the Weierstrass points stabilizing all the points in the complement of $U_1 \cup \{x_1, x_2\}$, as required by Lemma 2.1, ii), it is now enough to produce monodromy permutations which act transitively on $U_1 \cup \{x_2\}$, stabilizing the complement of this set, since then the group of monodromy permutations stabilizing the complement of $U_1 \cup \{x_1, x_2\}$ will act doubly transitively on $U_1 \cup \{x_1, x_2\}$, and will also contain the simple transposition (x_1, x_2).

To achieve this, we start again at $C'_{g-1} \cup Y_2 \cup Y_1 \cup E_1$ and smooth the restriction of L to $Y_1 \bigcup_{q_2} E_1$, to get an aspect L'_{E_1} which has ramification

$$\alpha := \alpha^{L'_{E_1}}(q_1) = \alpha^{L_{Y_1}}(q_1) = (g-3, g-2, \ldots, g-2, g-1).$$

This is possible, again, by dimensional properness. Keeping fixed the aspects $L_{C'_{g-1}}$ and L_{Y_2} already defined, the family

$$X = \mathring{G}^{g-1}_{2g-2}(E_1, (q_1, \alpha))$$

induces a family of limit canonical series on $C'_{g-1} \cup Y_2 \cup E_1$ whose monodromy stabilizes the Weierstrass points lying on C'_{g-1} and on Y_2, and which permutes the Weierstrass points on E_1, which are simply the ramification points other than q_1 of the series in X, so it is enough to show that the ramification points other than q_1 of the series in X are permuted transitively by the monodromy associated with the family X.

Now since E is irreducible, the points in the fibers of any finite map from E to another variety are permuted transitively, and it is thus enough to identify the sets of ramification points of series in X, other than q_1, with fibers of some map. This is done by the following result, which thus concludes the argument:

Proposition 2.2. *$X \cong \mathbb{A}^1$, and there is a map $E_1 \to \mathbb{P}^1$ of degree g^2 carrying q_1 to ∞ whose fiber over a point of $\mathbb{A}^1 = \mathbb{P}^1 - \infty$ is the set of ramification points other than q_1, with multiplicity, of the corresponding linear series in X.*

Proof. We may of course drop the $(g-3)$-fold base point at q_1, and identify X with

$$X' = \mathring{G}^{g-1}_{g+1}(E_1, (q_1, \beta)),$$

where $\beta = (0, 1, \ldots, 1, 2)$. Note that if $(\mathscr{L}, V) \in X'$, then the section in V vanishing to highest order vanishes to order $g+1 = \deg \mathscr{L}$, so $\mathscr{L} = \mathcal{O}_{E_1}((g+1)q_1)$ is constant, and X may be identified with a family of g-dimensional subspaces V in $H^0 \mathcal{O}_{E_1}((g+1)q_1)$ containing the section that vanishes to highest order, and meeting the (codimension 2) subspace V_2 of sections vanishing to order $\geqq 2$ in codimension 1. Since $\dim H^0(\mathcal{O}_{E_1}(g+1)q_1) = g+1$, the second condition on V implies that V *contains* the space V_2, and thus also implies the first condition. On the other hand, if $V \supset V_2$, then the ramification of $(\mathscr{L}, V)$ at q_1 is exactly β unless $V = V_1$, the space of sections vanishing at q_1; thus $G^{g-1}_{g+1}(E_1, (q_1, \beta))$ is the Grassmann variety of $g-1$-planes containing V_2, and $\mathbb{P}^1 \cong G^{g-1}_{g+1}(E_1, (q_1, \beta)) \supset \mathring{G}^{g-1}_{g+1}(E_1, (q_1, \beta)) \cong \mathbb{A}^1 \subset \mathbb{P}^1$ as claimed.

Consider the embedding of E_1 in $\mathbb{P}^g$ given by the complete series $|(g+1)q_1|$. The linear series in X' correspond, by the argument above, to the mappings given by projections of E_1 to $\mathbb{P}^{g-1}$ from points $v \neq q_1$ of the tangent line t at q_1 to E_1 in $\mathbb{P}^g$. The ramification points corresponding to projection from such a v are the points $q \in E_1$ such that the osculating $\mathbb{P}^{g-1}$ to E_1 at q in $\mathbb{P}^g$ contains v. This suggests passing to the dual curve $E_1^\vee \subset \mathbb{P}^g$ of osculating hyperplanes to E_1 in $\mathbb{P}^g$. Write $t^\perp$ for the $\mathbb{P}^{g-2} \subset \mathbb{P}^{g\vee}$ of hyperplanes containing t. The points $v \in t$ correspond to hyperplanes $v^\perp \supset t^\perp$, and ramification points corresponding to v are the points of $E_1^\vee \cap v^\perp$. Let $G: E_1 \to E_1^\vee$ be the Gauss map. Projection from $t^\perp$ to $\mathbb{P}^1$ induces, after removing base points if necessary, a map π, from the normalization E_1 of $E_1^\vee$ to $\mathbb{P}^1$. The ramification points away from q_1 corresponding to $v \in t$ are the points of the fiber over $v^\perp/t^\perp \in \mathbb{P}^1$, together, possibly, with the base-points other than q_1. It thus suffices to show that there are no base-points other than q_1, or equivalently that there is no point $q \in E$, $q \neq q_1$, whose image $G(q)$ is contained in $t^{\perp}$, or again that there is no such q such that the osculating hyperplane H to E at q contains t. But such an H would meet E at least g times at q and at least 2 times at q_1, a contradiction since the degree of E in $\mathbb{P}^g$ is $g+1$. This finishes the proof. □

3. Outline of an alternate proof for $g \geqq 5$

A proof involving fewer families and more projective geometry, which however only works for $g \geqq 5$, can be obtained as follows.

We proceed exactly as above for the arguments corresponding to Fig. 2.1 i)–ii). However, we replace Fig. 2.1 iii)–v) with a single degeneration of $C_{g-1} \bigcup_{p_1} E_1$ to $C''_{g-1} \bigcup_{p_1} E_1$, where p_1 is a dimensionally proper Weierstrass point of type $(0, \ldots, 0, 1, 1)$ on C''_{g-1}; these exist if and only if $g-1 \geqq 4$ by our (1987a) (whence the restriction to $g \geqq 5$). Since the C''_{g-1} aspect of a limit canonical series on $C''_{g-1} \bigcup_{p_1} E_1$ is unique, it suffices, by Lemma 2.1, to show that the monodromy of the family of E_1-aspects acts as the full symmetric group on the g^2+1 Weierstrass points of $C''_{g-1} \cup E_1$ lying on E_1.

By an argument similar to that of Proposition 2.2, the sets of such Weierstrass points are the hyperplane sections of the projection of the dual $E_1^{\vee}$ to an elliptic normal curve $E_1 \subset \mathbb{P}^g$ to $\mathbb{P}^2$ from the osculating $g-3$-plane in $\mathbb{P}^{g\vee}$ to $E_1^{\vee}$ at the image of q_1, together with any base-points away from q_1 that arise. Since the monodromy of hyperplane sections is well-known to be the full symmetric group, it suffices to prove that there are no base-points other than q_1, which may be done as in the proof of 2.2, and that the projection is birational. But birationality follows from the Plücker formulas which compute the ramification indices of the image of q_1 in $E_1^{\vee}$, since they show that the projection above to $\mathbb{P}^2$ and the "next" projection, to $\mathbb{P}^1$ from the osculating $\mathbb{P}^{g-2}$ to $E_1^{\vee}$ in $\mathbb{P}^{g\vee}$ at the image of q_1, have relatively prime degrees. □

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