Remembering David Buchsbaum

David Eisenbud and Jerzy Weyman

1. Career and Mathematical Accomplishments

David Buchsbaum earned his PhD at Columbia University in 1954 under Samuel Eilenberg. After postdoctoral sojourns in Chicago and Princeton, David spent most of his career at Brandeis University, and was very much engaged in building its mathematics department. He was elected to the American Academy of Arts and Sciences in 1995.

Though many PhD theses are forgotten or subsumed, the material in David's thesis [Buc55] remains essential knowledge. It contained the definition and an exploration of abelian categories, laying a general foundation for homological algebra. It was soon noticed by Grothendieck[1] and his colleagues, and employed in algebraic geometry. Now the notion of an abelian category is used in a vast area of modern mathematics.

David was famous early for his work with Maurice Auslander, who had been a fellow graduate student. Along with Serre's work on multiplicities, their work in commutative algebra was the first to show the power of homological algebra in that domain. Among their notable results was a formula relating depth and projective dimension and the characterization of regular local rings in terms of homological invariants [AB56], [AB57], [AB58], leading to the proofs that localizations of regular rings are regular and that regular local rings are factorial [AB59].

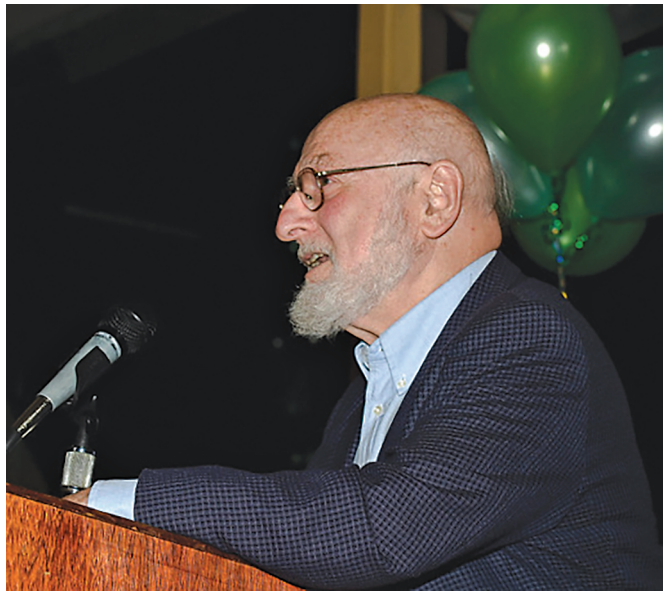

Figure 1. David in Berkeley, 2007.

Also highly influential was David's work with Dock Sang Rim on generic complexes [BR63], [BR64], [BR65].

David's love of Italy and close friendship with the Italian algebraic geometer/commutative algebraist Paolo Salmon had major repercussions for the Brandeis mathematics department; for a long time the department was enriched by a constant flow of bright young mathematicians from Italy to Boston who considered a year at Brandeis to be a normal and necessary part of their postdoctoral training.

David Eisenbud came to Brandeis in 1970 and soon started to collaborate with Buchsbaum on the structure of free resolutions. Their most quoted results were a

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[1] *David called them exact categories; Grothendieck is responsible for the adjective "abelian."*

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Figure 2. David in Tuscany, 2005.

Figure 3. David in New York, 1955.

characterization of the acyclicity of finite free resolutions [BE73] and a structure theorem for Gorenstein ideals of codimension 3 [BE77].

Nearly from the beginning of his career, David was interested in computing explicit minimal resolutions of determinantal ideals of generic grade. The "Buchsbaum–Rim" complex [BR63], [BR64], [BR65], later generalized by Buchsbaum and Eisenbud, extended work of Eagon and Northcott for maximal minors in an important direction. Alain Lascoux produced resolutions in characteristic 0 of all sized minors, using representation theory, but left some aspects vague. David and his students Kaan Akin and Jerzy Weyman systematized and completed the project [ABW82], also giving a characteristic-free version in the case of submaximal minors [ABW81]. Soon after that, Hashimoto proved that characteristic-free minimal resolutions for ideals of lower-order minors generally do not exist!

The Akin–Buchsbaum–Weyman approach to the submaximal minors was based on a characteristic-free version of Schur functors, and David subsequently became interested in the resolutions of Weyl modules in terms of tensor products of divided powers, which are projective modules over the Schur algebra. He continued to work on this project with Kaan Akin and they proved that the Schur algebra for general linear groups has finite global dimension—in particular, such resolutions exist [AB88]. The problem of finding explicit resolutions of Schur functors occupied David until the end of his mathematical activity, and remains unsolved to this day.

David formally supervised 24 PhD students, but an even larger group considered him a mentor, even a father-figure. We asked a number of these people to contribute memories for this article, and we hope you'll get a flavor of David's personal warmth by reading them. Aside from mathematics, David wrote many "letters to the editor" of *The New York Times*, which he collected in [Buc07b] and two collections of poetry, *Selected Poems* and *Algebra and Fire*. He also wrote a memoir, partly in poetry, [Buc07a] (Figure 2 is the cover picture of the memoir, half explaining its title *Through A Glass*). All are available on Amazon.

Figure 4. With Paolo Salmon in Turin, Italy, 2005.

Figure 5. With Mario Fiorentini and Giuseppe Valla in Turin, Italy, 2005.

David married his college sweetheart Betty, a writer and later a professor of English and published poet, when they were both 20. Betty and their three daughters, Helen, Susan, and Marion, have all contributed to this memorial. David was a devoted father, and got great pleasure from his grandchildren as well; one of them, Susan's son Gabriel Frieden, became a mathematician.

Figure 6. David and Betty in New York City, 1950.

2. Family

Betty Buchsbaum

David and I were married for 71 years. A marriage of opposites in many ways—he a mathematician, I a writer/poet. Yet a union that worked amazingly well. Many years ago, at a conference in David's honor, I attended one of the lectures. Unable to understand a word, I nevertheless was fascinated by the language. Whatever significance the terms had as metaphors in math, they had such different meanings in plain speaking English. The following poem was the result.

YOUR OTHER COUNTRY

Remember, years ago, a woman on a N.Y. train
turned on two young men talking math
non-stop—*You foreigners,* she fumed

either speak english or go back to your own country!
We thought it sad and funny, her discomfort
with math carried to xenophobic heights.

And I a young bride married to *one of those!*
But in truth I, too, found math forbidding.
It took years of living with you to see

your day in day out affair as close kin
to my passion as a poet. You'd work in a tent,
on a beach or train. All you needed

was pen and paper. In a pinch you'd do
with less, perfecting the skill of easing in
and out of solitude. Our daughters understood.

If you showed up early to drive them home
from parties, they'd say *Not yet Dad,*
you won't mind, just sit in the car and think!

Then the way you push the limits of the known.
For days, months, you play with a hunch,
let a premise lead you, without forcing it,

towards the as yet unseen, unheard . . . Face flushed,
you look for me in the house to say
you've *got it* and *it's beautiful!* By morning

you see flaws, try to simplify, make it elegant;
you, like a poet, speaking in metaphor.
Numbers *intertwine* like arms, legs, hearts;

a theory *tilts,* perhaps at risk; *loci,* sadly,
are of the *degenerate* kind; *ideals* tainted
by *duplicity;* and *quivers* can be *infinite.*

In-finite qui-vers—that titillates
my tongue. I'm not surprised you spawned
good work along those lines.

At a conference in your honor you summed up
your life in math: *I'm a man,*
you smiled, *of unresolved resolutions.*

Colleagues laughed, aware of problems
you hope to solve in the *field of resolutions.*
But I liked your deft way of saying

you're an ongoing paradox;
true, I might add, of our long marriage—
deeply familiar, yet strangely tantalizing.

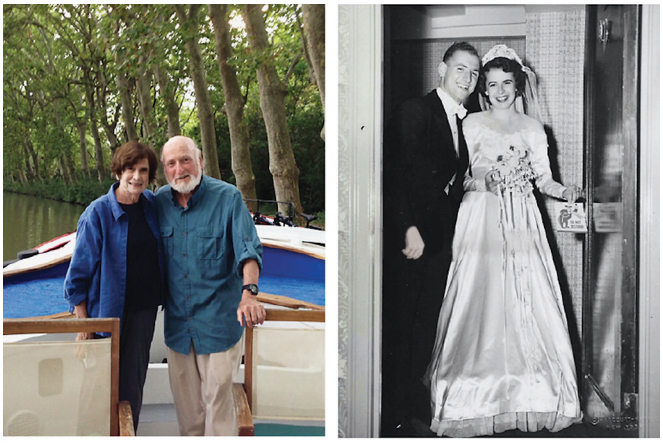

Figure 7. David and Betty in Westin, Massachusetts, 2016.

Figure 8. Betty and David in Newton, Massachusetts, 2016.

Helen Buchsbaum

Being intuitive and empathic may be antithetical to the stereotype of a mathematician's personality—yet captures my father. Early on he taught me the actual value of a numeric symbol depends on one's framework. Observing years of black squiggles on yellow pads, I witnessed symbolic representations of worlds I'd never see. I admired his passion seeking connections between systems not visible to the naked eye. And always felt seen by my dad for the person I am, not the one I might appear to be.

Susan Buchsbaum Frieden

Sometimes deep into the night when I was young, waking from a dream, I would make my way into my father's tiny study and onto his lap where he would cradle me in his left arm while scratching mathematics on a long yellow pad with his right. Lulled to sleep by the gentle motion of his fountain pen's squiggles, his pipe's floating smoke rings, his concentrated pleasure, I realise now that my father loving math was often my evening's lullaby.

Marion Buchsbaum

Growing up, I would sit in my father's treehouse-like office on the second floor of our home and discuss math with him at all hours of the night. I was a math major in college and my dad even came to my dorm the day before my first linear algebra midterm to help me prepare. His car got towed and he good-naturedly told me this gave us more time to discuss the material as we walked to the Cambridge Tow lot to rescue the impounded car. No memory of the exam, just of the bond we shared, and I treasure.

3. Students and Colleagues

Maria Artale

I met David Buchsbaum in Boston during my doctoral career as a graduate student. He was the advisor and guide for my doctoral thesis at Brandeis. During my stay there, and afterwards, I was able to appreciate his extraordinary humanity, his deep sensitivity, and his generous affection as a strong and benevolent father. Having an outstanding teacher like him was a precious gift that left a deep mark on my professional and human history, an experience shared by many of his students and colleagues. David was often a Visiting Professor at the University "Tor Vergata" of Rome. Many conferences and seminars in his honor have been

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organized in Italian universities. I will always remember him.

Giandomenico Boffi

I met David for the first time in 1977. It was at the University of Rome, during one of his frequent visits to Italy. It was before I went to Brandeis for a PhD program in Mathematics. I was impressed by his gentle manners and his fluency in Italian, as well as in several other languages. At that time I did not realize the special feeling between him and my country. Perhaps this feeling was partly based on the memory of Italian American children with whom he was friends in New York in the 1930s, as he told me many years later.

My strong bond with David was forged when he was my PhD adviser at Brandeis University. This bond also extended to cultural and personal issues. In addition to talking about resolutions of determinantal ideals and Schur and Weyl modules or complexes, we used to discuss the characteristics of our two countries, politics and religion.

After I got my PhD in 1984, David and I kept in regular contact for mathematical and non-mathematical reasons. Our families became friends and for my children he was like a dear relative living a bit far away. However, we did not write any mathematics together until 2004. David and I had a language arrangement: we would speak in Italian when in Italy, in English elsewhere; when exchanging emails, each one would write in his own language.

In the first half of 1998 I organized (with Alfio Ragusa, Elisabetta Strickland, and Giuseppe Valla) a two-month research event in David's honor, a very special event! It was mainly sponsored by INdAM, the Italian national institute of higher mathematics. The event attracted distinguished mathematicians from all over the world and took place in Catania, Rome, and Genoa. It was a tribute to David both as a mathematician and as a man, a tribute by the Italian mathematical community in recognition of the contacts David had maintained since the late 1960s.

The article David and I published in 2004 led to a larger project: a joint book that summarized a significant portion of David's mathematical interests. Partly based also on some lecture notes I had written at Brandeis University prior to my PhD, the book was published by Oxford University Press in 2006. We published another short paper two years later.

In the last dozen years we were not in contact as regularly as before, although we met a few times and exchanged occasional emails, and David gave me useful advice on some sensitive family matters.

Giandomenico Boffi is a professor at the University of International Studies, Rome.

Figure 9. Antony Geramita and Betty Buchsbaum.

I cannot say that David's passing came as a total surprise, given his age. But my children, my wife, and I have been deeply saddened. Forever grateful for all that he gave me and my loved ones, we offer our sincere prayers for him, for Betty, and for his entire family.

Corrado DeConcini

David Buchsbaum invited me to spend the spring semester of 1978 at Brandeis. In those years I worked in Pisa, but I was a frequent visitor to the department "Guido Castelnuovo" of the Univeristà di Roma (now called Sapienza Univeristà di Roma), where Buchsbaum was a familiar face; I must have met him in Rome when I was an undergraduate. He loved the city; he was "always in Rome."

I also spent the academic year at Brandeis in 1980–81. My recollections tend to mix up the two periods; the reader should forgive the confusion.

David told me that sometime at the beginning of the sixties Aldo Andreotti, a professor in Pisa, had invited him to spend some time there. On the way to Pisa, Buchsbaum passed through Rome to give a talk. He fell in love with the city, and Rome became an important part of his life.

The trip to Brandeis was my first visit to the United States. When my wife and I arrived in Boston, the city was covered by a couple of meters of snow, a lunar view for us since in Roma it snows only once every ten years. We lived in Cambridge and every morning I took the Boston and Maine train to reach the university. The first few days I was a bit lost, but with the help of the two Davids (Buchsbaum and Eisenbud) I very quickly felt at home.

At that time, David and his two students Akin and Weyman were working on the resolutions of determinantal

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Figure 10. With Betty Buchsbaum and Mitsuyasu Hashimoto.

ideals. Lascoux's thesis, which introduced plenty of combinatorial stuff into the subject, had just appeared, and they were trying to extend Lascoux's work to a characteristic-free setting. They eventually succeeded for the submaximal minors but, as we now know from Hashimoto's counterexample, there is no characteristic-free minimal resolution of determinantal ideals in general.

I vividly remember going to Buchsbaum's office, which was impregnated by the aroma of the tobacco of his pipe. I learned from him (and from David Eisenbud) the secrets of the so-called Buchsbaum–Eisenbud multipliers and the properties of the variety of complexes. I never really collaborated with Buchsbaum, though I wrote some papers with Eisenbud. The visits to Brandeis provided very great enrichment for my mathematics.

But there was not only mathematics. Buchsbaum was, in my eyes, the typical New York liberal intellectual. I learned a lot from him about American politics and literature. For example he gave me the book *A River Runs Through It and Other Stories* by Norman Maclean, a long time before it was published in Italy. Dining at the Buchsbaums' with Betty and David and some Italian commutative algebraist passing through Boston, and a 1990 Thanksgiving party with all of the Buchsbaum family and my baby son Guglielmo are memories I cherish.

In 1998, I went to three different conferences to celebrate David's birthday, two in Rome and Genoa and one in Boston.

Both my wife and I were born in Rome and lived all of our lives there. We passionately love our city, but this is nothing compared with the passion and love David, and later Betty, felt for Rome. When the spring came, David came. In the first years he came alone and stayed in a hotel near the Pantheon. I walked to see him sometimes with Giandomenico Boffi, one of his two Roman students (the other was Maria Artale). We would pick David up and take a walk discussing the characteristic-free representation theory of $GL(n)$.

Later he started coming with Betty and the mathematical discussions decreased. They rented some of the most peculiar apartments in the centre of Rome—one of them was part of an abandoned church. The pleasure of taking a walk with David or having dinner in a small square in Trastevere or Campitelli did not go away and even increased over the years.

I have tried to communicate how dear and important David has been in my life. I shall miss him enormously.

David Eisenbud

David and I met in 1968 at the first conference I ever attended. I was a wide-eyed graduate student, and my advisor, Saunders MacLane, told me beforehand that I should pay special attention to the series of lectures that David was scheduled to give. At the conference I volunteered to be his scribe and write up his lectures, which began with the Koszul complex and the characterization of regular local rings. (I reused the ideas of that exposition in my own book on commutative algebra.) At the end of the series, David spent a couple of lectures riffing on his more recent work, related to resolutions of determinantal ideals. I found his exposition impressionistic, and brashly proposed a different organization. We finally agreed to leave that material out entirely!

By that time I knew that I would like to work with David again. When I got my PhD, I chose Brandeis over Yale because of his magnetism.

At Brandeis, David was my mentor, and eventually my collaborator in a particularly productive and happy period of my life. He also became a dear friend. We met Saturdays at Brandeis, or in the basement of his house, and over many hours we worked on free resolutions. Schur functors were just beginning to be important to us then, and at one point we decorated the wooden frame of the blackboard in David's basement with a formula for them, to help (mostly me) remember the notation.

David nursed and protected me through the stress of the tenure process. He remains for me a model of good mentoring!

David introduced me not only to mathematical ideas, but also to department and university politics, which he took very seriously. I learned a lot about the tensions within the department and their origins, and also about the efforts by David and others that had led to the remarkable rise of the Brandeis department between 1960 and 1970 (when I joined). What I learned from him in this way has been very helpful to me in navigating

Figure 11. With Bernd Ulrich, Luchezar Avramov, Frank-Olaf Schreyer, and David Eisenbud at Berkeley, 2007.

administrative tasks, first at Brandeis and then at MSRI. David loved to travel, most of all to Rome. He was a mentor to students and postdocs from all over the world, but especially in Italy. Many Italians arranged to spend time at Brandeis to be near him. Through his connections, a small area of Brandeis was even (briefly) declared to be a part of Italy, so that an important collection of Italian manuscripts could be housed there without violating Italian law. In honor of his contribution to Italian mathematics, David's birthday was celebrated in 1998 with a six-week conference spread over three Italian cities.

David communicated his great love of mathematics as an integral part of his life and culture. He shared deep intellectual interests with Betty, his wife for more than 70 years, a poet and professor of English, who survives him. David and Betty were very close to their three daughters, Helen, Susan, and Marion, and their warm family life has always seemed to be a shining example. David told me with great pleasure of his conversations about mathematics with Susan's son, Gabriel Frieden. Gabriel has followed David into mathematics, and is currently a postdoctoral student in Montreal, thinking about some of the same sort of mathematical problems that fascinated his grandfather.

Gabriel Frieden

For me, mathematics will always be intertwined with the memory of my grandfather. One of my earliest mathematical memories is from second or third grade, when I told him that "zero is not a number, but a place holder," as I'd been taught in school. He took exception to this, insisting that zero is just as good a number as one, two, or three. Although it would be several years before I understood why he felt so strongly about this, the idea that numbers have their own identity and intrinsic value made an immediate impression on me. This idea was reinforced through birthdays: when he turned 72, he excitedly explained that this is the only age in a human's lifetime with a prime factorization of the form $a^b \cdot b^a$.

After I moved away for college, one of the highlights of each trip home to Boston was meeting with Grandpa to discuss the new ideas that I was most excited about. During graduate school, our mathematical relationship slowly shifted towards me being the teacher, with exact sequences and complexes giving way to Young tableaux and crystal graphs on the pages of his yellow legal pads. When I started to make progress in my research, his interest gave me confidence, as well as the first opportunity to shape my results into a coherent story. His infectious enthusiasm for mathematics will always remain with me.

Mel Hochster

I have many memories of David Buchsbaum over the decades—he was a kind, gentle man to whom family was very important. He was always ready to help junior mathematicians progress with their work. His terrific research has always been a source of great inspiration for me. But I will focus here on three personal interactions with him that have stayed with me for decades.

When I was going through a period (which admittedly lasted for a good chunk of my life) when I was constantly giving incorrect proofs of the direct summand conjecture (or the existence of big Cohen–Macaulay modules) in mixed characteristic, on several occasions I called David to tell him my latest fable. Even when I interrupted his dinner, he always listened graciously, and when I eventually called back (this was before the advent of email) to tell him that the idea was wrong, he managed to keep me from feeling bad about wasting his time.

His mathematics was wonderful, and his mental agility was amazing. On one occasion I was giving a talk in the Boston area about work which was not fully written up yet. Someone in the audience asked a question about a rather important detail that I had not thought through. I went numb. David rescued me—even though he was hearing about what I was doing for the first time, he immediately saw the needed explanation of the missing detail, a subtle variation on a homotopy argument, and gave an eloquent explanation.

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Mel Hochster is the Jack E. McLaughlin Distinguished University Professor at the University of Michigan.

On the way back from a conference in England, David and I spent a day together in London. We had dinner in a mediocre Chinese restaurant and then went to see the play *Amadeus*. Our seats, obtained at the last minute, may have been the worst in the theater. But with David's company I remember this as one of the most enjoyable evenings I have ever spent.

I will miss him greatly.

Craig Huneke

My first contact with David was when I visited Brandeis in the winter and spring of 1977. There I sat in my first (and only) commutative algebra class as a graduate student. The instructor was David Buchsbaum, and the course was on homological algebra. I was in awe of him, after having already studied many of his classic papers, particularly his papers with Maurice Auslander, and his (at that time recent) work with David Eisenbud. I still remember some of the topics in the course, which influenced me a great deal, particularly his discussion of Koszul complexes, and questions about their homology. Another strong memory from that class was him writing a huge multilinear complex across the entire blackboard, covering one wall of the room, and then saying something to the effect of "This is what we all learned in high school." (Not in my high school!!)

I got to know David much better when we both stayed in the same dorm in Bonn during a several-month visit in the spring and summer of 1980. It was a real joy to listen to his many insights and his history during our dinners and pub visits. It was a great time for me.

After Bonn, it was easy for me to speak with him, and we interacted quite a bit for several years. This had one drawback—at that time he was an editor for the *Journal of Algebra*, and I received many, many referee requests from him! (He once sent me a hand-drawn card for the "referee of the year" because I had refereed so much.)

Commutative algebra would not be the same without him, but I miss most his easy-going friendship, and how great he was to be around.

Daniel Ruberman

When I arrived at Brandeis in the mid-eighties, I met David as part of the group of distinguished mathematicians on the faculty at the time. David was in his mid-fifties, still very active in studying representation theory, and very much a part of department life. David felt intense pride in the department that he and others had built from scratch in the late 1950s and early 1960s, and took pains to inculcate me and others of succeeding generations into this recent but strong tradition. He often talked about the history of the department and about colleagues over the years, hoping to convey the deep sense of commitment to the enterprise that kept him and others at Brandeis in spite of opportunities to go elsewhere.

In quiet ways, David took me under his wing and taught me much about being a department citizen and how to fight for our interests in a university that was still working on how to match its ambitions and finances. I became chair as he was preparing to retire, and I found him to be a generous mentor and great resource in dealing with issues both inside and outside of the department. He had strong principles, and coupled those with great empathy and insight into other people. David's interest in the growth of his junior colleagues was noted by many of us who came of age in the department.

In addition to the great impact his mentorship had on me, I also have a lasting impression of David's infectious sense of humor and irony. His eyes would light up at a good line, even (or especially) if it was one of his. Once, when I was chair, I came to the department over Christmas break and found David in his office. I proudly told him that I'd seen the Dean at the gym, and sealed the deal on a long-sought approval for a new position. With a twinkle in his eyes, David gestured to an old wooden squash racquet gathering dust in a corner. "Yes," he said, "that's the way it gets done. You didn't think we really liked playing squash all those years," and burst into laughter.

Our mathematical interests didn't overlap much—David would say that while he'd made his mark in algebra, he had always wished to "do something geometric." But he taught me a great deal about being a mathematician, and about how to stay active in research by keeping an open mind for new ideas. I admired his zest for life (and his enthusiasm for all things Italian) as much as I admired his mathematics, and very much miss his presence.

Rafael Sanchez

I knew about David Buchsbaum long before I went to Brandeis in the summer of 1982. I had read several articles written by him and had decided to do my PhD at Brandeis University under his guidance. Being his student, and later a colleague and friend, was a great life experience. Not only did I learn mathematics from him, I also learned about life, especially the importance of friends and family. David had a passion for Italy, which we shared and enjoyed during several work internships in Rome. At the time of the

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Figure 12. With Rafael Sanchez and Hema Srinivasan.

celebration of his 70th birthday in Genoa, in addition to an excellent conference in his honor, we walked through Le Cinque Terre, with a previous day of beach and gastronomy in Camogli. His visits to Caracas, the Central University of Venezuela, and the Venezuelan Institute for Scientific Research were varied and very productive. David was a teacher and a friend and I will always carry his memory and his teachings with me. He will be present every time I enjoy a glass of Barolo with gorgonzola cheese.

Hema Srinivasan

The most important thing I learnt from David was not mathematics but the attitude towards mathematics and what it is, and a clearer understanding of how one enjoys it. The hours I spent in David's office at Brandeis discussing problems, trying to do them on the board, and most of the time not getting anywhere, were some of the happiest moments for me. It has been many years since I left Brandeis, but our discussions of mathematics, politics, and philosophy have remained as cherished memories.

Hema Srinivasan is a professor at the University of Missouri.

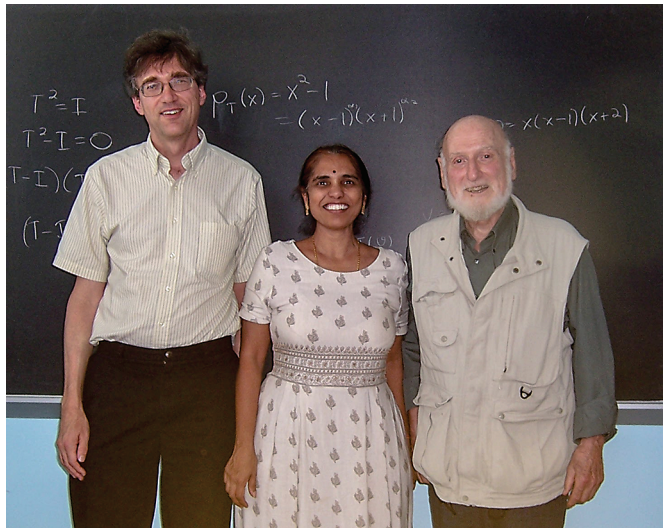

Figure 13. With Dale Cutkosky and Hema Srinivasan.

Richard Stanley

I started to be interested in the connections between commutative algebra and combinatorics around 1972 when I was at Berkeley. In 1973 I moved to MIT. Fortunately at that time, the world center for commutative algebra was just a stone's throw upstream at Brandeis University.

Among the luminaries at Brandeis was David Buchsbaum. It was a great pleasure to have David as a mathematical resource. He was always upbeat and enthusiastic about any topic we discussed, mathematical or not. I had regarded homological algebra as an excessively formal and dry subject until I was exposed to David's work (much of it joint with Maurice Auslander) on the connections between commutative algebra and homological algebra. I was especially enthralled by the famous Auslander–Buchsbaum formula $\mathrm{pd}(M) + \mathrm{depth}(M) = \mathrm{depth}(R)$, where R is a commutative noetherian local ring and M is a nonzero finitely-generated R-module of finite projective dimension. This fundamental result (in the context of graded algebras rather than local rings) plays a key role in my own work, for example in the characterization of Cohen–Macaulay face rings by Gerald Reisner.

In 1985, both David and I paid our first visit to Asia to attend the US-Japan Joint Seminar on Commutative Algebra and Combinatorics in Kyoto. We were on the same flight to Osaka. I sat further to the front of the plane than David, so I could boast to him that my first time in Asia was about 0.03 seconds earlier than his. We were both greatly impressed by the legendary Japanese hospitality.

The main direct connection between David's work and combinatorics concerns his characteristic-free approach

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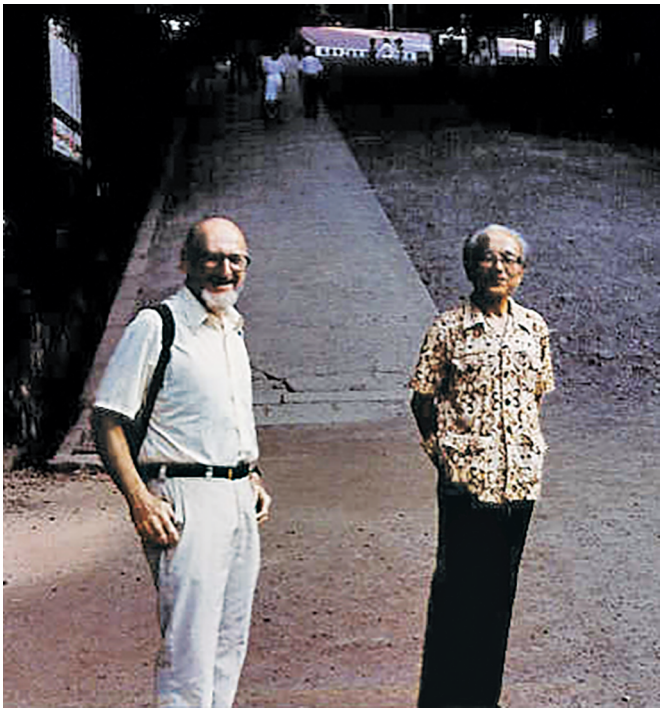

Figure 14. With Hideyuki Matsumura.

to the representation theory of GL(n). Kaan Akin was his primary collaborator in this work, but it also included a nine-year collaboration with Gian-Carlo Rota on the characteristic-free projective resolutions of Weyl modules. Nowadays much of this work can be regarded as a characteristic-free categorification of some fundamental identities, such as the Jacobi–Trudi identity, from the theory of symmetric functions. It shines an interesting light on a major topic within algebraic combinatorics.

Elisabetta Strickland

David Buchsbaum had a real penchant for Italy, as if something in his blood was absolutely in tune with it. He visited Rome so many times that, at my University "Tor Vergata," we called him "our permanent visitor."

The first time I had the pleasure of listening to one of his talks was in 1978: my enthusiasm for his ideas in representation theory was overflowing and the first chance I had I went to Boston to spend a semester at Brandeis University where David was. I arrived in the States just after a blizzard, and the mornings of my first days were difficult: To go to Brandeis I first had to clean the path from the snow in front of the house my husband (Corrado DeConcini) and I rented from Barry Mazur in Cambridge. We then took the Boston and Maine railway to Waltham. It smelled of lobsters, as its main activity during the day was to carry huge containers of the crustaceans.

When I finally reached my office, I spent pleasant hours working in a nice room close to David's. Time passed very quickly, listening to the seminars at the "Fellowship of the ring" or watching David walk up and down in front of the blackboard in his room, explaining his ideas and filling the space with complicated complexes. He loved to write down incredible numbers of arrows that represented the maps that he had created overnight!

Once I visited him at home on Thanksgiving: I'll never forget what he and Betty, his wonderful wife, had invented to help us understand what that day represented for Americans. That was my first encounter with cranberry sauce and sweet potatoes, and I loved them. Betty wrote fascinating poetry: over the years I collected the lovely books she wrote, which came to Rome in David's suitcase; each time it was a wonderful gift.

When I asked David to read my mathematics his comments were precise and useful, and he was always interested and patient. He loved art, music, and literature. In Rome, he visited the museums and enjoyed the beauty of the city. Among the apartments where he stayed was a deconsecrated church transformed into a flat, where he slept in the apse. We often walked the narrow streets near Piazza Navona and Campo dei Fiori, his favourite spots. New complexes took form in his mind just watching the blue sky over the roofs.

At one time there was an Iraqi student at my university in Rome. This was during the war in Iraq, and he occasionally disappeared to Baghdad because someone in his family had been hit by a bomb and lost an arm or a leg or had died. Maria Artale and I were his mentors. I asked David to be his external advisor. The case was desperate, but with David's help the student obtained his doctorate and now is a professor in the University of Baghdad. Thank you David, I'll never forget your patience and generosity.

The last time I saw David, he was crossing Piazza S. Maria in Trastevere, helping himself with a walking stick. I saw that the end of an era was near—our wonderful permanent visitor would soon have to give up.

Paolo Valabrega

I was introduced to David Buchsbaum in 1971 by my advisor, Paolo Salmon, a very close friend of David's. Salmon strongly recommended that his students visit Brandeis for a period of at least one year after completing their Italian degrees. I took that advice. While I was at Brandeis, I had a chance to attend David's beautiful homological algebra

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Figure 15. With Paolo Valabrega and Eugene Gover.

class. The perspectives that he provided opened my eyes. Before that time, I had studied commutative algebra, but without homological methods. The usefulness of the new approach was made very clear to me in discussions with David on a variety of homological and commutative topics. This led me to interests that, while new, were still connected to what I had learned before from Paolo Salmon.

Our relationship was mainly mathematical at the beginning, but over the years it developed into a truly close friendship, which included our wives and also Paolo Salmon. We shared many interests: politics, current events, poetry, Judaism and the State of Israel, languages, and letters to newspapers (which he often wrote, sometimes with a copy to me). He had a great love for Italy, for its artistic beauties, its food, cuisine, and its language.

I remember many occasions when we got together in Boston, Torino, or Catania, but especially in Rome where we used to have dinner together with our wives at the restaurant Monserrato, next to Piazza Navona. I also remember a pleasant trip to Camogli with a final stop at a wonderful "focaccia" bakery. David's favorite cheese was gorgonzola, which my wife and I were ready to provide each time he visited us in Torino.

The famous theorem of Auslander–Buchsbaum is very well known (at least among algebraists), but not many know that before they had found their final proof, Nagata was working to find a counterexample. Commenting on this situation, David said: "Auslander and I worked very hard at the proof, in order to arrive before Nagata's counterexample!"

After 2014 our health prevented us from meeting and we switched to email. We agreed that he would write in English and I would write in Italian, thereby giving each of us the opportunity to practice the other's language. But three or four years ago, David, who at the time was taking an advanced Italian language class, asked me to allow him to write in Italian, with the supplementary request that I correct his style and usage. This was really quite easy because his Italian was exceptionally good, even elegant, and sometimes very sophisticated.

His knowledge of Italian was quite surprising, and he was proud of it. For example, the word "ammiccare" (winking) appeared with an odd definition in an Italian crossword puzzle many years ago in Genova. At a roadside picnic, several Italian mathematician friends were struggling with the clue. David overheard what was going on and, smiling, casually gave the answer.

Our common languages included Hebrew as well as Italian and English, and we discussed words and idioms in all three, thus combining politics and linguistics. I also recall David's (incomprehensible to me) conversation in Yiddish at our dining room table with a Polish friend of ours.

Paolo Salmon rejected the use of the internet, and after he retired in 2000, he stopped having direct contact with David. Thereafter, he and David used my emails in order to communicate with each other.

David's passing is a big loss for me and for all of his friends.

Jerzy Weyman

David was my mentor, in many ways a role model, and a very dear friend.

I met David in January 1978 when I arrived as a graduate student at Brandeis. We immediately started discussing problems related to determinantal ideals and Schur functors. I was extremely lucky because at that time David was getting interested in representation theory and I had the opportunity to learn it from him. Kaan Akin was another of David's students and the three of us spent the next two exciting and enjoyable years thinking about these subjects. The three of us wrote two papers: one on the construction of Schur complexes and one on characteristic-free resolutions of ideals of submaximal minors of generic matrices. This collaboration became a model for me.

After leaving Brandeis I diverged mathematically from David: I turned to more geometric problems, while David went on working with Kaan. They wrote some beautiful papers, and proved that the Schur algebra of the general linear group has finite global dimension. This result was later extended to other classical groups by Donkin. The characteristic-free resolutions of Schur functors became the main focus of David's later work, which included a collaboration with Gian-Carlo Rota.

Five years after my thesis I was back in the Boston area at Northeastern University. I went to Brandeis every Wednesday to attend the Fellowship of the Ring seminar. Before or after the seminar I would talk with David about mathematical and non-mathematical subjects: US and international politics, science, music, art, and life in general. This continued for many years.

Later, when I moved to Connecticut, I would visit the Boston area often and every time I would stop at David's

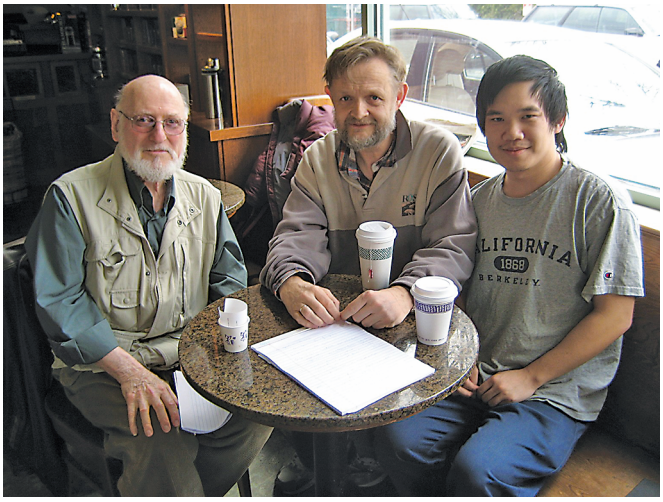

Figure 16. With Jerzy Weyman and Steven Sam.

place for some nice conversation. Some of these visits coincided with my dentist appointments, so often David would ask me jokingly by email about my next dentist appointment, to make sure the date was available. He did not do mathematics at that time but he was still interested in new developments in the area of free resolutions.

In 2016, David and Betty invited me and my girlfriend to visit, and we spent a wonderful weekend at their place. At that time I met all his daughters and some grandchildren.

In the fall of 2019, I was invited to Betty and David's 90th birthday celebration. It was a great opportunity to see his family, and to learn many new things about him.

In the fall of 2020, there was a small seminar on free resolutions run remotely by David Eisenbud and myself. David (Buchsbaum) would sometimes connect to listen to our discussions of recent progress, and sent me some comments. He seemed to enjoy it very much.

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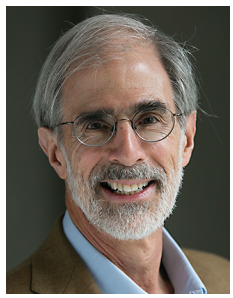

David Eisenbud

Jerzy Weyman

Credits

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