ERRATA FOR THE PRACTICE OF ALGEBRAIC CURVES

DAVID EISENBUD AND JOE HARRIS

p. 9, Theorem 0.3: We should have added the hypothesis that $X \cap Y$ is generically smooth.

As stated, this version of Bézout's theorem can fail when the intersection scheme $X \cap Y$ is not generically smooth. For example If X is the union of two 2-planes meeting in a point in $\mathbb{P}^4$ and Y is a generic 2-plane passing through this point, then the degree of $X \cap Y$ is 3, not 2 as asserted. (This example can be massaged to make X into a variety).

However the generically smooth case implies the correct result $\deg([X][Y]) = \deg(X)\deg(Y)$, where $[X][Y]$ is the product in the Chow ring.

p. 17 line -11: "it is a" should be "it corresponds to a"

p. 25, line 16: "the map $C \to \phi(C)$ is finite". Correct, but we should have mentioned "quasifinite & projective implies finite" in the prerequisites section.

p. 30, line 4: ÷ prints as the sign for dividing one number by another. Should print as Div .

p. 31, line -3 : after the summation sign, dx_1 should be dx_0

p. 36, line -3: $(\mathcal{O}_{\mathbb{P}^1}, W)$ -¿ $(\mathcal{O}_{\mathbb{P}^1}(d), W)$

p. 49: line -4, formula for p_a: K_X should be K_S

p. 50, Theorem 2.8: Delete the +1 at the end of the formula.

p. 53, This is not correct. The correct version is given in exercise 2.14. Delete exercise 2.2 (2).

p. 61, line -1,-2: delete the (incorrect) phrase beginning ", so Γ imposes... ."

p. 65, line -9: $H^0_*(I_C/Q)$ should be $H^0_*(\mathcal{I}_C/Q)$

p. 65, line -5 : I_C/Q should be $\mathcal{I}_C/Q$

p.72: "sum of $\mathrm{Sym}^{e_i} V$" . It would be better to say "a sum of representations $\mathrm{Sym}^d V$ for various d (the symbols e_i were used for the basis of V).

p. 78, line 17: "normal sheaf" -¿ "conormal sheaf"

p. 78, line -10: "is a globally on C" -¿ "is a global section of ω_C."

p. 82, part (2) of the proof of Corollary 4.8. Replace this paragraph (which is nonsense) by:

(2) In this case a divisor Γ in the series $\mathcal{V}$ fails by 2 to impose independent conditions on forms of degree $d-3$. It follows that if we subtract a point from Γ the resulting divisor Γ' fails by at least one to impose independent conditions on forms of degree $d-3$. Thus Γ has degree at least d, and by

part (1), Γ' is contained in a line. Repeating this argument with another point of Γ, we see that Γ must be the intersection of C with a line; that is, $\mathcal{V}$ is the series $|\mathcal{O}_C(1)|$.

p. 83, paragraph beginning on line -7. What is written is insufficient; the degree of the polynomial equation could conceivably be 0. Here is a characteristic 0 argument: We may assume $n > 1$. By the Riemann-Roch theorem, the linear series $|np|$ has no basepoints, so the corresponding map $\phi : E \to \mathbb{P}^{n-1}$ is well-defined of degree n, and if $n > 2$ then ϕ is an embedding.

If ϕ were not surjective, it would be constant; equivalently, $np \sim nq$ for every pair of points p, q. If $n = 2$, this would mean that then every point would be a ramification point of ϕ and in characteristic 0 this is impossible, for example by [1, Theorem 7.11a].

We note that the surjectivity of $[n]$ holds in positive characteristic as well; for a characteristic-free proof, see for example [2, Corollary 6.4] which proves the finiteness.

p. 90: In the skew-symmetric matrix B, the top 2 rows should be

$$\begin{pmatrix} 0 & 0 & x_0 & x_2 & x_3 \\ 0 & 0 & x_1 & x_3 & x_4 \end{pmatrix}$$

and the first two columns should, similarly, be the negative transpose of this matrix.

Also, delete the reference to [Eisenbud 1995, Theorem 11], as I'm no longer sure what was intended.

p. 90: The result of Exercise 4.1 is the same as Corollary 4.8.

p. 90: Exercise 4.3. the x_0^2 is multiplying the wrong side of the equation.

p. 91: The statement of the Sylvester Gallai theorem should be: In any finite set $\Gamma \subset \mathbb{P}^2_{\mathcal{R}}$ not contained in a line there is a line containing just 2 points of Γ.

p. 97, last sentence of 2nd para of 5.6: "$\mathrm{Pic}_d(C)$ is a for $\mathrm{Pic}_0(C)$ should say "$\mathrm{Pic}_d(C)$ is a torsor for $\mathrm{Pic}_0(C)$, which means that if $\mathcal{L}_0$ is an invertible sheaf of degree d on C, then the map $\mathrm{Pic}_0(C) \to \mathrm{Pic}_d(C)$ sending $\mathcal{L} to \mathcal{L} \otimes \mathcal{L}_0$ is an isomorphism."

p. 104, line -15: end of line, should be ≥ 4. Better argument for this para: If D is a special divisor , then $|D|$ is a subseries of the canonical series, which itself is not very ample.

Theorem 10.1: n should be r.

p. 106, third line before subsection 5.8: Should be $\omega_C \otimes \mathcal{L}'^{-1}$, where $\mathcal{L}'$ has degree 1

p. 107: Lemma 5.17: change $\mathbb{P}^r$ to $\mathbb{P}^n$ to avoid the conflict of notation between that r and the superscript on Σ^r_d.

p. 107, next to last sentence before the Proof of Martens Theorem: Should say: we see that $\Sigma_d^{r-1} \smallsetminus \Sigma_d^{r-1}$ is dense in Σ_d^{r-1}.

p. 107 line -7 Marten's should be Martens'

p. 108 Exc 5.2: This is the wrong defn of "freely". Should be: $gx = x$ only when g is the identity.

p. 108 Exc 5.3.2 : A^2 should be $\mathbb{A}.^2$.

p. 114. This whole discussion assumes that the "topological covering space" is a finite covering.

p. 142, line 4: m should be m_0.

p. 143, Example 7.14: the wording is confusing. Better: In the Grassmannian of lines $\mathbb{G}(1,3)$, the Plücker coordinates of the line L that is the span of the points q, r are the 2×2 minors of the matrix..."

p. 149, line 2: "locally" should be "locally analyitically" (or étale locally?) on B in order to find these sections. So in the end, you are using descent to "glue" together the morphism.

150: Second para of the proof: It might be necessary to make a base change before there are sections sections of your family of genus 1 curvesThis is OK since to construct an isomorphism of the pullbacks of the families after finite base change.

p. 153 In constructing $Hilb^\circ$, the curves should be smooth, irreducible and non-degenerate.

p. 187: The proof given for the case of equality in Castelnuovo's theorem is not correct: the top line of the first displayed diagram is NOT left exact.

To fix it, redraw the diagram, eliminating the top row, and replacing m by k.

Let a_k denote the right-hand map in the middle row of the diagram, and let b_k denote the right hand map in the bottom row of the diagram. Equality in Castelnuovo's theorem implies that for all k the rank of a_k, which is the number of conditions imposed on V_k by Γ, is equal to the rank of b, which is the number of conditions imposed on $H^0(\mathcal{O}_C(k))$ by Γ

Since the bottom row of the diagram is left exact, the rank of b is $h^0(\mathcal{O}_C(k)) - h^0(\mathcal{O}_C(k-1))$. Since V_{k-1} is contained in the kernel of a, the rank of a is $\leq \dim V_k - \dim V_{k-1}$, whence

$$\dim V_k - \dim V_{k-1} \geq h^0(\mathcal{O}_C(k)) - h^0(\mathcal{O}_C(k-1))$$

for every k. The rest of the proof can be taken starting with "For large m the restriction map..."

p. 201: last para before 11.3. can intersect... should be cannot intersect....

p. 202: We are rather cavalier about the fiber product, and this can be confusing.

p. 203, statement of Thm 11. delete first half of 2nd line (it's a broken, repeated phrase).

p. 212: Cor 12.5. Hypothesis should be $r \geq 1$.
p. 213: $C_{d-1} = \binom{2d-2}{d-1}/d$ is the correct number.
page 224, between the first two displayed equations: "and the on C to be" presumably should be "and the *ramification divisor* on C to be"
page 242, line 7: "an" should be "on"
page 242, last displayed equation: $d+1$ should be d

REFERENCES

[1] David Eisenbud and Joe Harris. *3264 and all that: a second course in algebraic geometry*. Cambridge University Press, Cambridge, 2016.
[2] Joseph H. Silverman. *The arithmetic of elliptic curves*, volume 106 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1986.

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF CALIFORNIA AT BERKELEY, BERKELEY, CA 94720, USA
Email address: de@msri.org

DEPARTMENT OF MATHEMATICS, HARVARD UNIVERSITY, CAMBRIDGE MA 02138
Email address: harris@math.harvard.edu