ERRATA FOR THE PRACTICE OF ALGEBRAIC CURVES

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p. 9, Theorem 0.3: We should have added the hypothesis that $X \cap Y$ is generically smooth.

As stated, this version of Bézout's theorem can fail when the intersection scheme $X \cap Y$ is not generically smooth. For example If X is the union of two 2-planes meeting in a point in $\mathbb{P}^4$ and Y is a generic 2-plane passing through this point, then the degree of $X \cap Y$ is 3, not 2 as asserted. (This example can be massaged to make X into a variety).

However the generically smooth case implies the correct result $\deg([X][Y]) = \deg(X)\deg(Y)$, where $[X][Y]$ is the product in the Chow ring.

p. 17 line -11: "it is a" should be "it corresponds to a"

p. 25, line 16: "the map $C \to \phi(C)$ is finite". Correct, but we should have mentioned "quasifinite & projective implies finite" in the prerequisites section.

p. 30, line 4: ÷ prints as the sign for dividing one number by another. Should print as Div .

p. 31, line -3 : after the summation sign, dx_1 should be dx_0

p. 36, line -3: $(\mathcal{O}_{\mathbb{P}^1}, W)$ -¿ $(\mathcal{O}_{\mathbb{P}^1}(d), W)$

p. 49: line -4, formula for p_a: K_X should be K_S

p. 50, Theorem 2.8: Delete the +1 at the end of the formula.

p. 53, This is not correct. The correct version is given in exercise 2.14. Delete exercise 2.2 (2).

p. 61, line -1,-2: delete the (incorrect) phrase beginning ", so Γ imposes... ."

p. 65, line -9: $H^0_*(I_C/Q)$ should be $H^0_*(\mathcal{I}_C/Q)$

p. 65, line -5 : I_C/Q should be $\mathcal{I}_C/Q$

p.72: "sum of $\operatorname{Sym}^{e_i} V$" . It would be better to say "a sum of representations $\operatorname{Sym}^d V$ for various d (the symbols e_i were used for the basis of V).

p. 78, line 17: "normal sheaf" -¿ "conormal sheaf"

p. 78, line -10: "is a globally on C" -¿ "is a global section of ω_C."

p. 82, part (2) of the proof of Corollary 4.8. Replace this paragraph (which is nonsense) by:

(2) In this case a divisor Γ in the series $\mathcal{V}$ fails by 2 to impose independent conditions on forms of degree $d-3$. It follows that if we subtract a point from Γ the resulting divisor Γ′ fails by at least one to impose independent conditions on forms of degree $d-3$. Thus Γ has degree at least d, and by

part (1), Γ' is contained in a line. Repeating this argument with another point of Γ, we see that Γ must be the intersection of C with a line; that is, $\mathcal{V}$ is the series $|\mathcal{O}_C(1)|$.

p. 83, paragraph beginning on line -7. This is nonsense. Here is a characteristic 0 argument: We may assume n¿1. If the map p-¿ np were not surjective, it would be constant; equivalently, np nq for every pair of points p,q. But then every point would be a ramification point of the map corresponding to the complete series —np—, so this map would be inseparable.

p. 90: The result of Exercise 4.1 is the same as Corollary 4.8.

p. 90: Exercise 4.3. the x_0^2 is multiplying the wrong side of the equation.

p. 91: The statement of the Sylvester Gallai theorem should be: In any finite set $\Gamma \subset \mathbb{P}^2_{\mathcal{R}}$ not contained in a line there is a line containing just 2 points of Γ.

p. 97, last sentence of 2nd para of 5.6: "$\mathrm{Pic}_d(C)$ is a for $\mathrm{Pic}_0(C)$ should say "$\mathrm{Pic}_d(C)$ is a torsor for $\mathrm{Pic}_0(C)$, which means that if $\mathcal{L}_0$ is an invertible sheaf of degree d on C, then the map $\mathrm{Pic}_0(C) \to \mathrm{Pic}_d(C)$ sending $\mathcal{L} to \mathcal{L} \otimes \mathcal{L}_0$ is an isomorphism."

p. 104, line -15: end of line, should be ≥ 4. Better argument for this para: If D is a special divisor , then $|D|$ is a subseries of the canonical series, which itself is not very ample.

Theorem 10.1: n should be r.

p. 106, third line before subsection 5.8: Should be $\omega_C \otimes \mathcal{L}'^{-1}$, where $\mathcal{L}'$ has degree 1

p. 107: Lemma 5.17: change $\mathbb{P}^r$ to $\mathbb{P}^n$ to avoid the conflict of notation between that r and the superscript on Σ^r_d.

p. 107, next to last sentence before the Proof of Martens Theorem: Should say: we see that $\Sigma^{r-1}_d \setminus \Sigma^{r-1}_d$ is dense in Σ^{r-1}_d.

p. 107 line -7 Marten's should be Martens'

p. 108 Exc 5.2: This is the wrong defn of "freely". Should be: $gx = x$ only when g is the identity.

p. 108 Exc 5.3.2 : A^2 should be $\mathbb{A}.^2$.

p. 142, line 4: m should be m_0.

p. 143, Example 7.14: the wording is confusing. Better: In the Grassmannian of lines $\mathbb{G}(1,3)$, the Plücker coordinates of the line L that is the span of the points q, r are the 2×2 minors of the matrix..."

p. 187: The proof given for the case of equality in Castelnuovo's theorem is not correct: the top line of the first displayed diagram is NOT left exact.

To fix it, redraw the diagram, eliminating the top row, and replacing m by k.

Let a_k denote the right-hand map in the middle row of the diagram, and let b_k denote the right hand map in the bottom row of the diagram. Equality in Castelnuovo's theorem implies that for all k the rank of a_k, which is the number of conditions imposed on V_k by Γ, is equal to the rank of b, which is the number of conditions imposed on $H^0(\mathcal{O}_C(k))$ by Γ

Since the bottom row of the diagram is left exact, the rank of b is $h^0(\mathcal{O}_C(k)) - h^0(\mathcal{O}_C(k-1))$. Since V_{k-1} is contained in the kernel of a, the rank of a is $\leq \dim V_k - \dim V_{k-1}$, whence

$$\dim V_k - \dim V_{k-1} \geq h^0(\mathcal{O}_C(k)) - h^0(\mathcal{O}_C(k-1))$$

for every k. The rest of the proof can be taken starting with "For large m the restriction map..."

p. 201: last para before 11.3. can intersect... should be cannot intersect....

p. 202: We are rather cavalier about the fiber product, and this can be confusing.

p. 203, statement of Thm 11. delete first half of 2nd line (it's a broken, repeated phrase).

Remark 12.6 (p. 213): the Catalan number is one off; it should be $\binom{2d-2}{d-1}/d-1$

p. 212: Cor 12.5. Hypothesis should be $r \geq 1$.

p. 213: $C_{d-1} = \binom{2d-2}{d-1}/d$ is the correct number.

page 224, between the first two displayed equations: "and the on C to be" presumably should be "and the *ramification divisor* on C to be"

page 242, line 7: "an" should be "on"

page 242, last displayed equation: $d+1$ should be d

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